OUR STORY ABOUT SOLVING STANTON MAA PROBLEM 5
(aka, placing 3 items on 25 place grid)

I did this problem with my 14-year-old son, who considers math to be his worst subject, and his 17-year-old friend, who is very strong in math.

I created a sheet with the problem and the steps that Stanton uses (Have an emotional reaction, What is the problem asking?, Can you draw a picture? Can you solve a smaller version of the problem?, etc.) with blank spaces inbetween for them to work out their solutions.

My son reads the problem and has his emotional reaction: “I have no idea. I don’t even understand what the problem is saying.”

Meanwhile, his friend starts working out the entire problem in his head. “Hmmmm....that means there would be 25 options for the first piece, 16 options for the second piece, and 9 options for the third piece....”

Me (thinking): “Man, this is incredible. He is working it out just as Stanton eventually walked us through in his paper. Except it took me going over it about 3 times to understand what he was doing WHILE he was explaining it...”

Back to friend: “So that’s easy if the question is asking about permutations. Is it asking about permutations or instances?”

Me: “Uhhhhhhhh......” (thinking: PERMUTATIONS? INSTANCES? I don’t think I have those words in my Stanton cheat sheet.....)

Friend (clearly noting I have no idea how to answer his earlier question): “Does it matter in what order they are? Or it is just how many separate position combinations are possible?”

Me: “Uh, well, no, it’s not different orders of the same pattern. Each pattern is just counted once.”

Friend: “OK, well, that makes it a bit more difficult. Then I guess we have to divide by 3 factorial.”

Me (thinking): “THREE FACTORIAL! I KNOW that’s not in my cheat sheet. What is a factorial again......?”

Me (aloud, knowing that he is supposed to divide by 6 from said cheat sheet): “So what is 3 factorial?”

Friend: “6.”

Son (piping in for the first time): “Don’t you know that? EVERYBODY knows that 3 factorial is 6.”

ME (thinking): “Gee, thanks, son. At least he refrained from say ‘DUH’.”

Son continues his emotional response by ranting on again about one of his ongoing math complaints that he doesn’t think that 0 factorial should be 1. He brings this issue up on a recurring basis, and so is glad for yet another opportunity to insist that it shouldn’t be that way.

Meanwhile, friend announces: “OK, so that would be 600.”

Which is the answer. But so far, not one thing has gone down on the paper. And, of course, the problem-solving technique is supposed to be solving a smaller version of the problem first, which obviously was not necessary for 17-year-old friend.

So, Me thinking: “NO! You did it wrong! It was too easy! What about a smaller version?”

But Me saying: “That’s fantastic! Great job, Friend. Son, did you understand what Friend did with this problem.”

Son, of course, did not, having been distracted during the calculation phase by his 0 factorial rant.

Me: “Friend, can you explain it to Son.”

Friend patiently tries to explain to Son, but Son is not understanding the basic concept of the whole problem.

Friend: “So, how many options are there to place the first circle?”

Son: “Uh, 25?”

Friend: “Right. Then how many options are there to place the second circle?”

Son: “25.”

Friend: “No, no...”

Son: “Oh yeah, I see. 24.”

Friend: “No, because it can’t be in the same row or column as the first one....”

Son: “Huh?”

Repeat several times.

Now for my only substantive contribution to the problem: “OK, Son, think of it this way. Imagine you have a chess board with 25 squares....”

Son: “Chess boards have 64 squares...”

Me: “Fine, imagine a MINI chess board with 25 squares.”

Son: “OK.”

Me: “So now imagine that you have three rooks. What you are trying to do is figure out how many separate ways you could place those three rooks on the board so that none of them could capture the other ones.”

Son, brightening: “OK.”

Friend, meanwhile, has been reading the piece of paper for the first time. Now he is scratching his head. “Solve a smaller version of the problem? Huh? I don’t know... I’m just going to solve the problem my way.”

Me: “That’s fine. Go ahead.”

Friend (now starting to draw the problem): “So say we put the first one here.” (Draws on the grid and colors out the squares in that row and column.) “How many squares does that eliminate?”

Son: “10”

Friend: “No.”

Son: “5 in the row and 5 in the column.”

Friend: “No, no.”

Me: “Honey, you are counting the same square twice. You can’t count the square both on the row and on the column.”

Son: “Oh, right. 9.”

Friend: “Right, 9. So how many squares does that leave as options for the second piece?”

Son: “16.”

Friend: “Right. That means there are 25 options to place the first piece, and 16 options for the second piece for each of those options.”

Son: “Okay.”

Friend: “So that is 25 x 16. But now we have to figure out about the third piece.” (Draws a second piece and colors off that row and column.) “How many places does that leave for the third piece?”

Son: “9.”

Friend: “Right, for each of all those options there are 9 options for the third piece. So that is 25 x 16 x 9.”

Son: “Okay.”

Me: “Do you want to use the calculator on my iPhone?”

Son: “Nah.”

Me (resisting rolling my eyes and thinking): “Yeah, like my son is going to be able to figure that out in his head....”

Friend: “But that equation gives all the permutations of the three pieces. But we don’t care about the order--we only want to count the same position once.” (Draws three circles and labels them A B C.) “So, this counts ABC and BCA and CAB and CBA and...and BAC...and ACB. But because they are all the same position, we only want to count them once.”

Son: “Yeah.”

Friend: “So that means we have to divide by 6.”

Son: “Yeah.”

Friend: “So we have 25 x 16 x 9, but 6 doesn’t go into any of those evenly. But if we divide 9 by 3 we get 3, and if we divide 16 by 2 we get 8, so that’s 8 times 3, which is 24. So then we have 25 x 24. But if we split 24 into 4 x 6, then we can multiply 25 by 4 to get 100, then 100 times 6 is 600.”

Son: “YEAH!”

Me: Speechless and astounded.

Me: “So what did you get on your SAT scores?” (Just kidding! I didn’t really say that.)

I did praise both the boys about working through the problem so well, and also told Friend what a great job he did explaining the problem. I told him, truthfully, I couldn't have worked it out nearly as well as he did.

So there you have it--The Good, the Bad, and the Ugly (the Ugly mostly being ME). Lessons learned? Well....

The way the boys did it, we never got to the problem-solving technique we were supposed to be doing, which was #5: Solve a Smaller Version of the Same Problem. But they did use #4: The Power of Drawing a Picture, and #9: Avoid Hard Work.

This isn't one of the official Problem-Solving Techniques, although Maria talks about it a lot herself. But making a story out of the problem, or relating it to real-world items with which he was familiar was necessary for my son to make sense of the problem.

Friend is pretty awesome at mathematical problem solving.

Son is easily intimidated, but stuck with it and got the problem once his friend explained it. So to some extent, that demonstrated #7: Perserverance is Key.

It was really fun to see the boys doing math together.

And how come I'm the only one who doesn't know that 3 factorial is 6? Does everyone else walking around out there just automatically know that?

Cheat sheet needs some additional vocabulary for math-impaired moms to use when working with students proficient in math.