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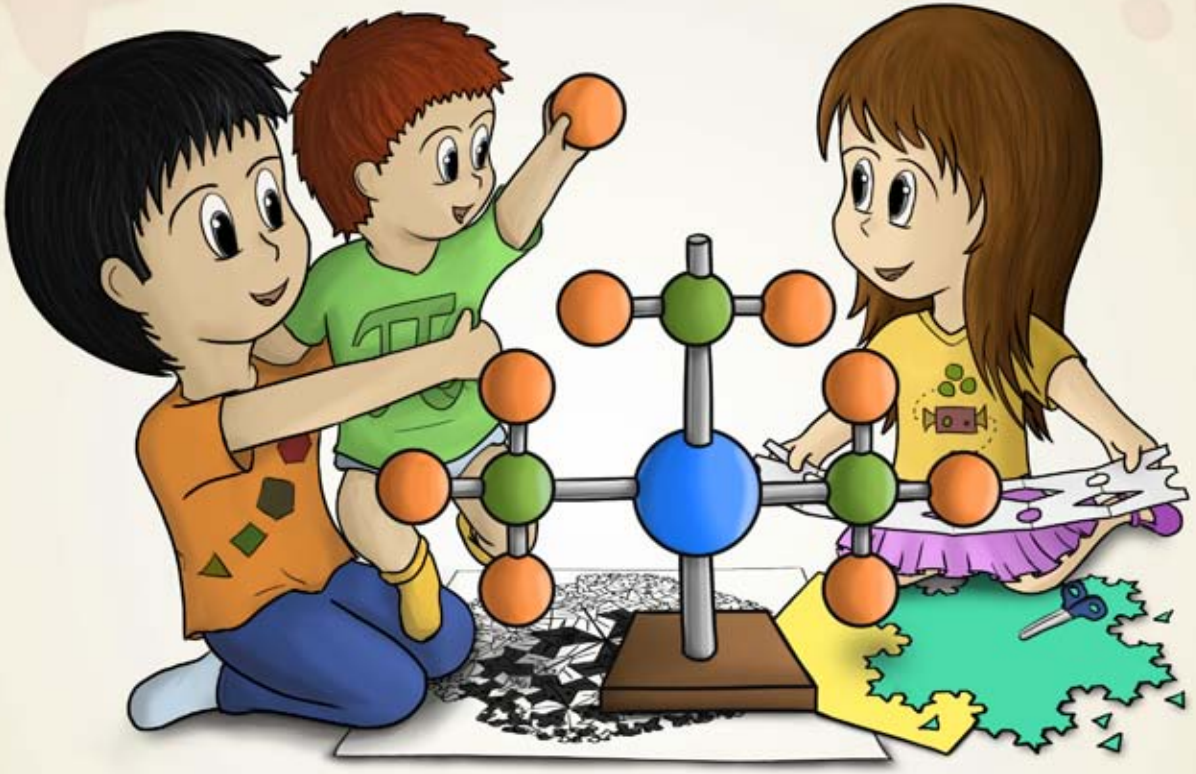

Adventurous math for the playground crowd

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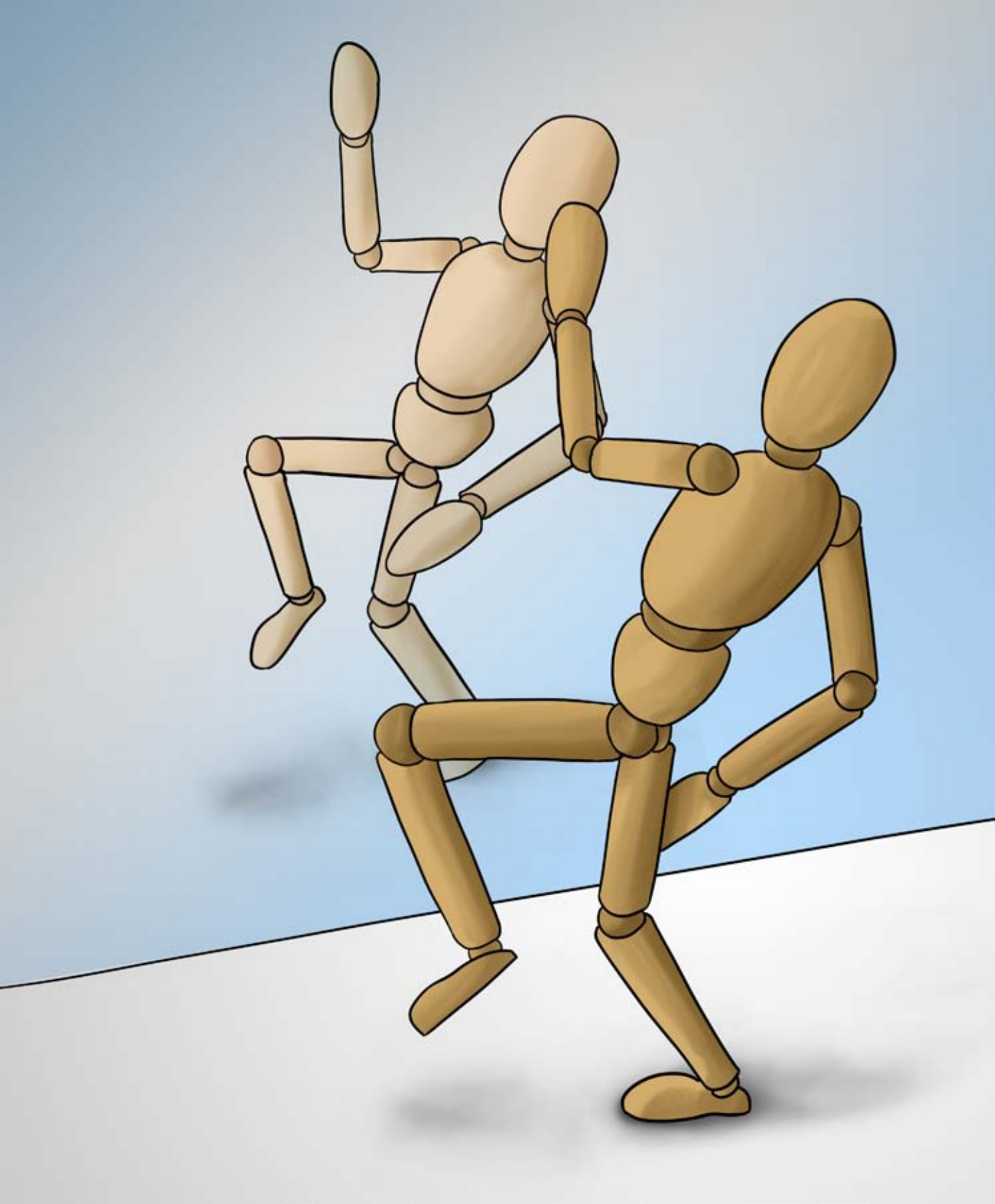
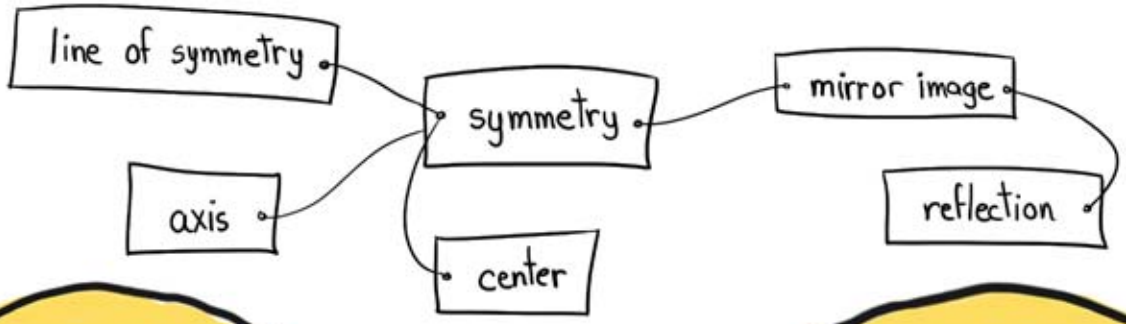


Live mirror

Stand in front of each other and mimic each other's gestures and expressions. That's it! Sounds too easy? As soon as you try crisscross poses, asymmetric finger shapes, or fast motions, the game provides enough challenge even for an adult! You can easily adjust the difficulty to match each child's gross and fine motor skills, as well as attentiveness to details.



Many dances have parts for two people mirroring one another. Put some music on and compose a dance as you play the game. Choreographers break the symmetry to add tension and drama!

For a hardcore challenge, draw two perpendicular lines and play the game with four people. Expect much confusion! This is just like playing Special Snowflake games, where you and your friends are the snowflake.

Play live mirrors when your child's day is not going right, you can't stop arguing, or a homework session is not going well. A sympathetic human mirror gives an immediate boost to the child's thinking abilities and emotional well-being! This happens, in part, because brain's special "mirror neurons" fire when we see an action performed by another person.

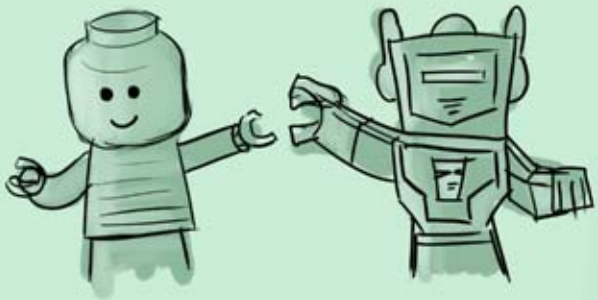
Find finger positions and interesting movements that challenge children. Help kids who get confused by mimicking them in return, or gently positioning their limbs with your hands. Ponder why some motions are harder to mimic than others.

Let your baby lead. Follow by mirroring the baby's gestures and facial expressions. Holding the baby in your lap, mirror someone else's gestures by moving baby's hands or feet.

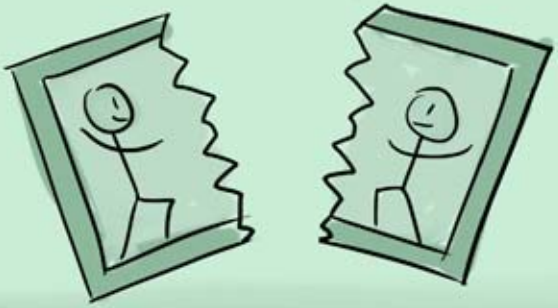
Offer whole body or limb movements, rather than fine gestures. Help with more challenging movements by positioning your child. You can also help by telling the story of your movements using math words: up/down, forward/backward, front/back, perpendicular, across, and so on. Avoid using "left/right" because it confuses in the context of mirrors.

Try more complicated movements – for example, rub your tummy and pat your head. Experiment with friends using two or more "mirrors" (that is, lines of symmetry).

Instead of your own bodies, manipulate dolls, plush toys, models made from construction sets, or posable action figures. It's a math lesson taught by LEGO®, a Barbie®, and a Transformer®!



Take pictures of your games – this will inspire kids to strike more interesting poses! If you take pictures from the side, with the symmetry line in the middle, you can cut them in half and play a matching game with the pieces.

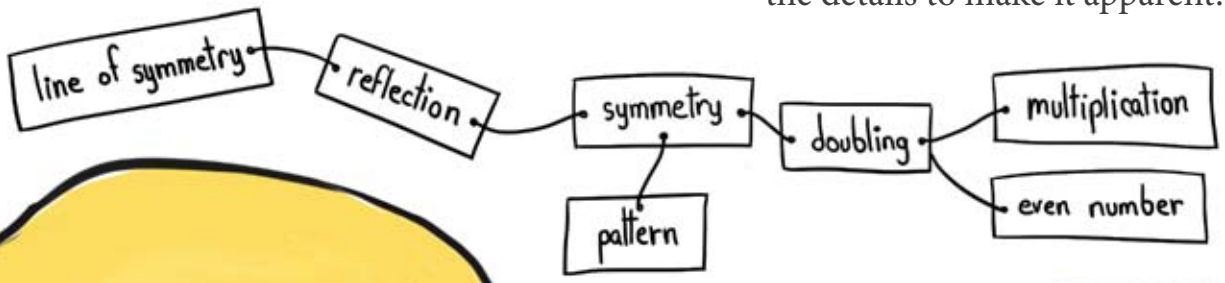


Add objects to the game – give each player a ball, a hula hoop, a large wooden block, or something you can climb. Objects can help kids notice and discuss the idea that "my right is your left."

Go on scavenger hunts for characters striking mirror poses or doing copycat movements in games, cartoons, paintings, or dances. Directors, choreographers, and artists use symmetry to express both harmony and conflict. For example, mirrored confrontation shots often appear on posters for action movies.

Double doodle zoo

Fold a piece of paper in two. Make a random doodle in such a way that it overlaps the fold line – otherwise, the result will fall apart. Now comes the fun part: invite your game partner to see something in the doodle! Add features to it and draw details to make the image more apparent. Cut out the result and open it. Turn the paper over, see something else in the opened shape, and draw the details to make it apparent!



Use the same doodle with different paper folds and predict what it will look like depending on the type of symmetry. Cut out the doodle and check the accuracy of the predictions.

Introduce some randomness into the game – doodle with your eyes closed.

Make it delicious – bake a fairly large and complex cookie (e.g., gingerbread man, butterfly) and try to eat it one bite at a time while preserving the symmetry.

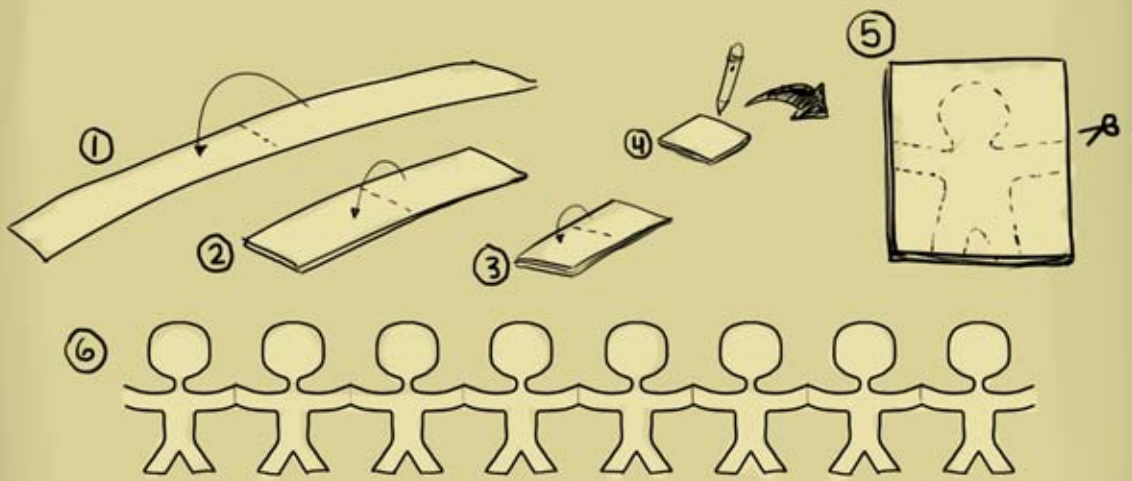
Observe kids growing their math eyes as they notice more and more key points, angles, symmetries, quantities, and other mathematical features. Pay attention to these features yourself when it is your turn to see something in the doodle, say their names out loud, and otherwise encourage looking for properties.

Parents can play this game by themselves and give the resulting paper toys to the baby. You can use the cutouts to make a mobile or a garland. A version for babies who can point or use signs: have a lot of pictures of toys, animals, and so on shown at once, and invite the baby to point or put the doodle next to the object it resembles.

Toddlers will be more successful drawing on large sheets of paper. Many toddlers can see animals, houses, cars, or other familiar shapes in clouds, tree bark, or abstract rug designs. Help toddlers add more details to their scribbles until something familiar emerges.

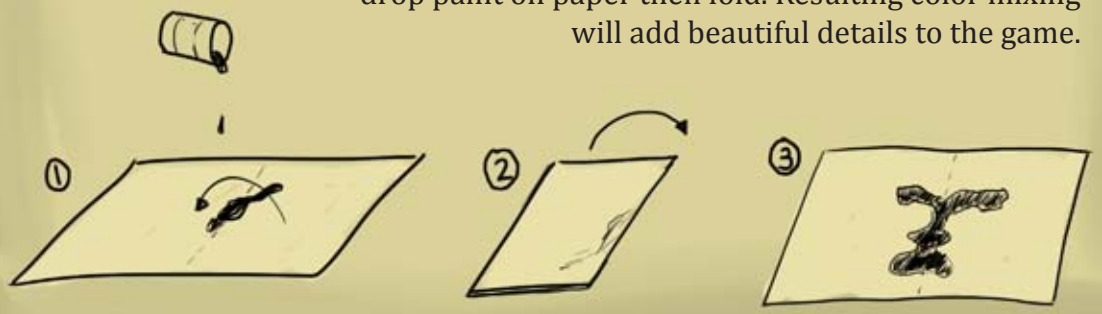
It's better to play the game quickly until you create many examples. Only through many examples do children develop property-noticing strategies. However, some older kids may want to tinker with a shape they particularly like, turning it into a more polished project, such as the Japanese art of Notan (positive-negative space).

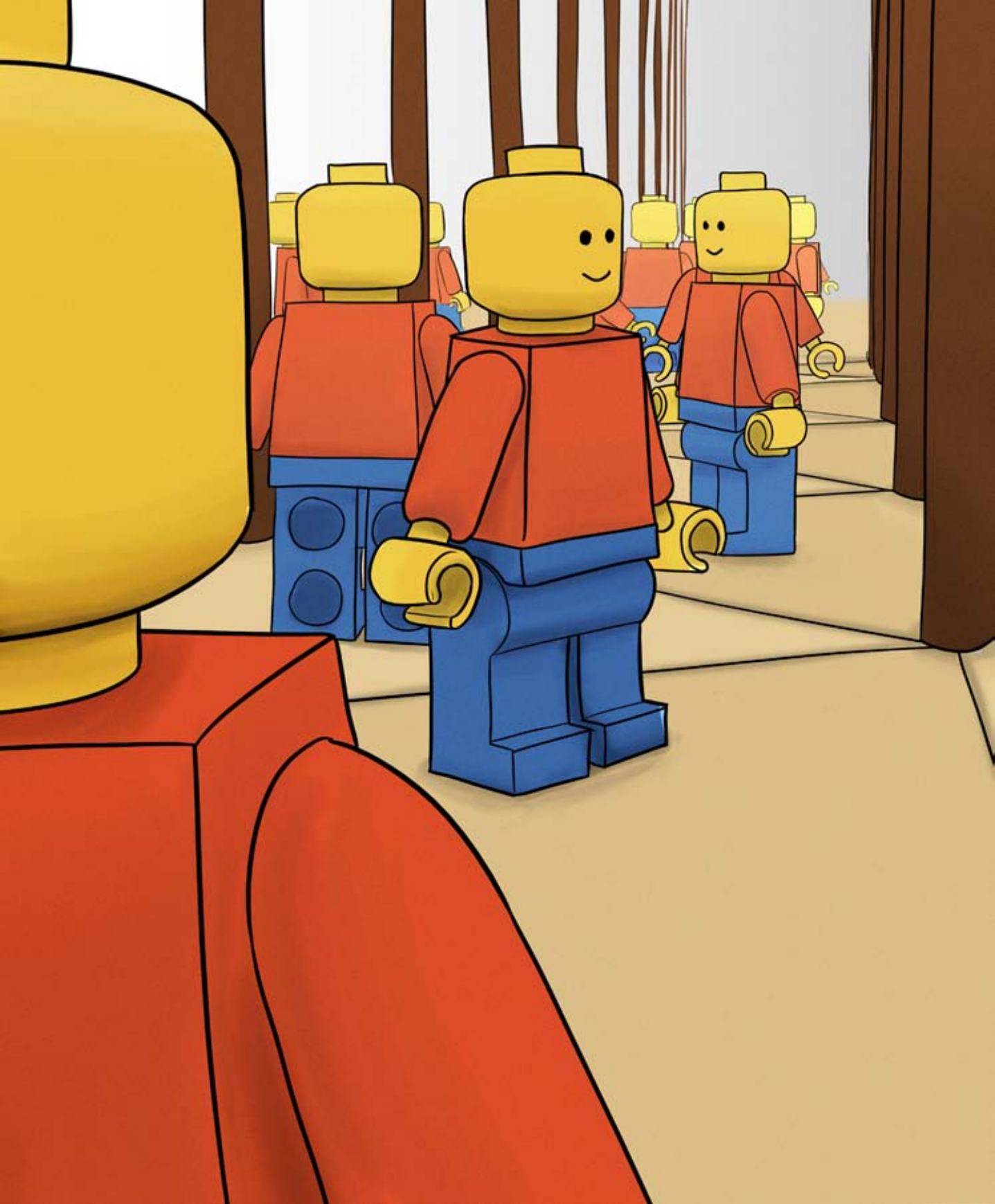
Fold the paper into more than two parts. Observe what changes.



Play this game in reverse – give a child a cutout of a shape or a doodle and suggest to fold along its line (or lines) of symmetry, if any.

Use paint instead of pencil or marker. Paint or just drop paint on paper then fold. Resulting color mixing will add beautiful details to the game.

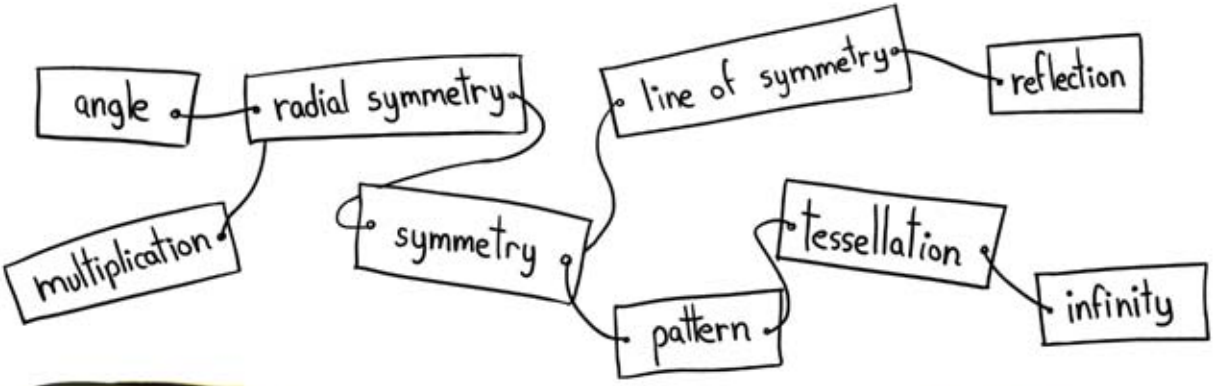




Mirror book

Lay two plain rectangular mirrors on top of one another, face to face. Connect them at one edge with duct tape. You can usually find small mirrors in dollar stores and school supply or craft stores, and larger mirror tiles in home improvement stores.

Put interesting things – like favorite toys or your face – inside the open mirror book. Change the angle between the mirrors and admire the greatest show in math!

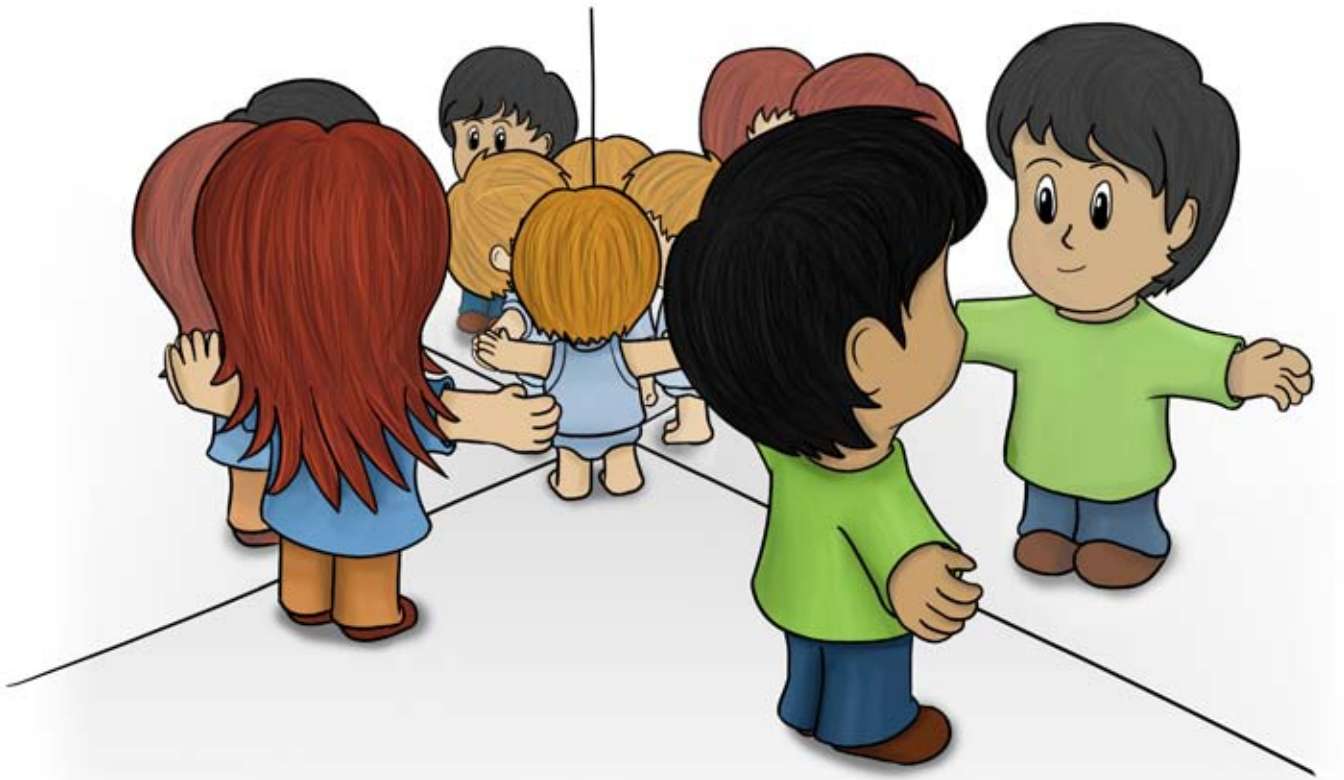


Prop the mirror book on a paper or whiteboard. Draw or write numbers, letters and words. Kids are often surprised and impressed by all the hidden symmetries they discover in the familiar shapes and symbols.

Kids find new artistic and scientific ideas when you combine the mirror book with the classic elements. For fire, place a candle, a flashlight, or a weak laser into the book. Half-submerge the book to compare reflections in the water and in the air. Place the mirror book on the ground in a park or a forest for a square-foot field trip close to earth.

Get a pizza slice and magically turn it into the whole pizza. Or try it with a slice of cake instead. See for yourself how math makes it possible to have your cake and eat it too!

As children play with the mirror book over days and weeks, observe how they make more and more accurate predictions of positions, angles, and shapes. Help children notice radial symmetry in nature: place the center of the mirror book over centers of live flowers, photos of starfish or radiolarians, or models of snowflake and crystals.



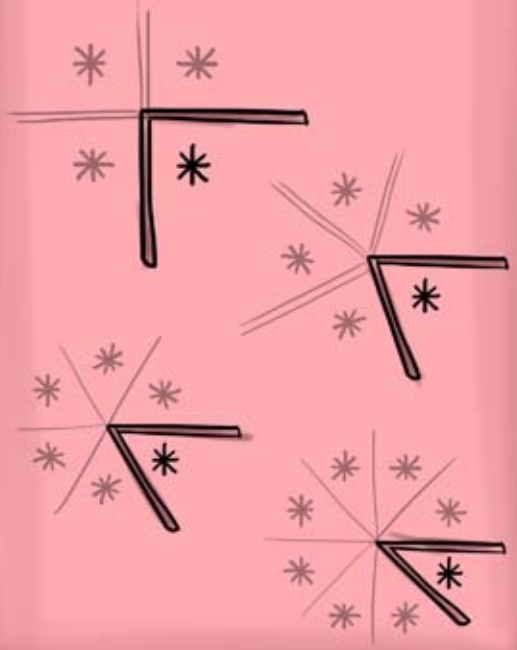
Put toys or your fingers inside the opened mirror book. Move the mirrors or the objects to make simple "animations" and tell stories. Move the mirror book over a baby's favorite picture book or family photos for funky kaleidoscope effects.

Help children experiment with drawing or sculpting inside the mirror book. Take turns posing simple puzzles – for example, "Can you make a square with one toothpick?"

Use one or more mirror books to design tiling patterns called tessellations: draw or make a fragment of the tiling inside the mirror book and observe the book taking it to infinity. Encourage your child to place or draw groups of objects inside the book – two shoes, three bears, four wheels – and observe what happens to the total number of objects and reflections when the angle between the "pages" of the book changes (multiplication tables).

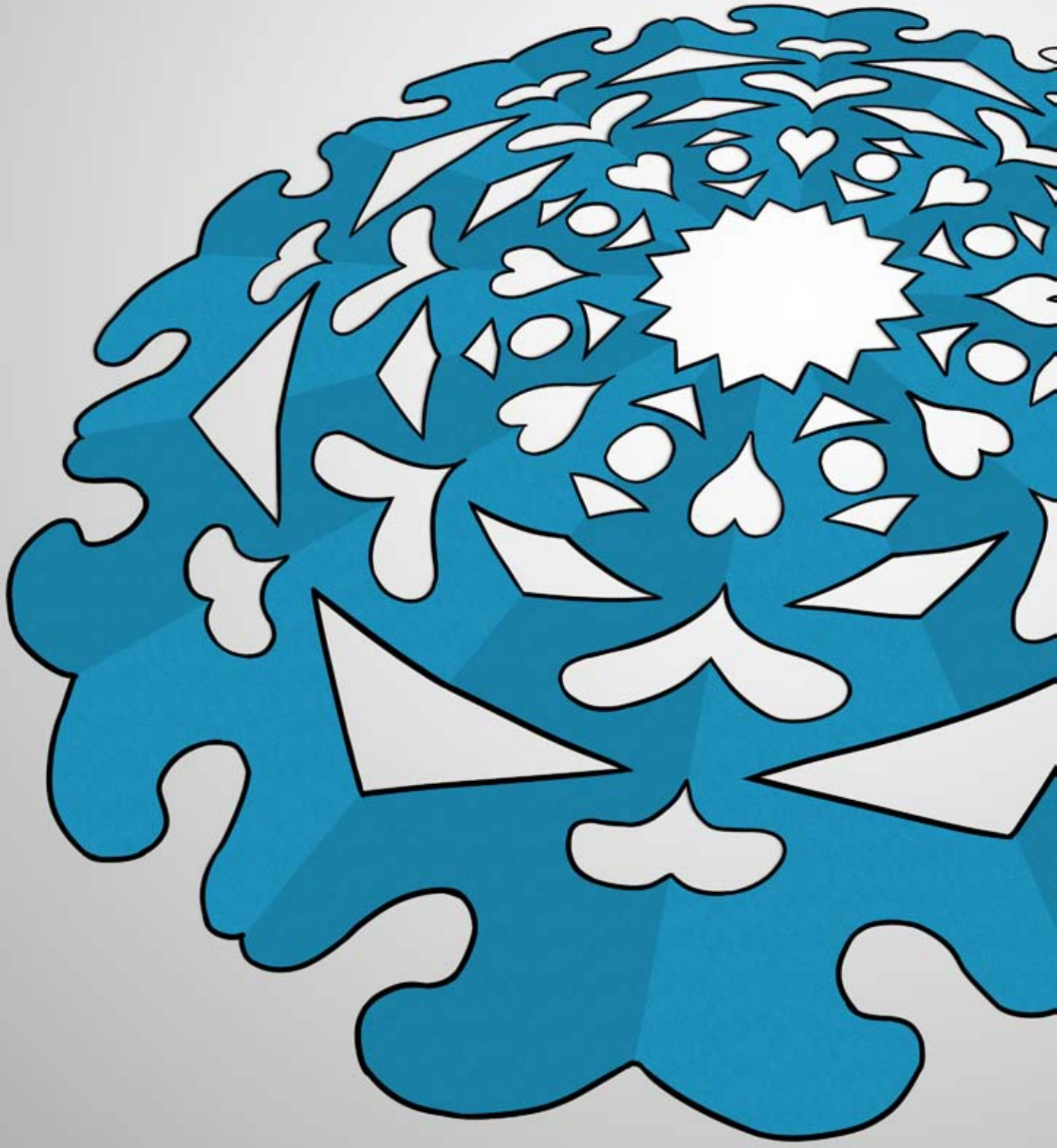
Put two mirror books next to each other to form a square “room” with mirror walls. Look inside from the top, to find infinity. Next, try a triangular mirror room – does it work the same?

Find all multiplication tables hidden in the mirror book. Open it ninety degrees for times four, seventy-two degrees for times five, sixty degrees for times six, or just count reflections. Even three-year-olds can intuitively find (if not necessarily measure) the angles that turn one toy into three toys, or four toys, in the mirror book.



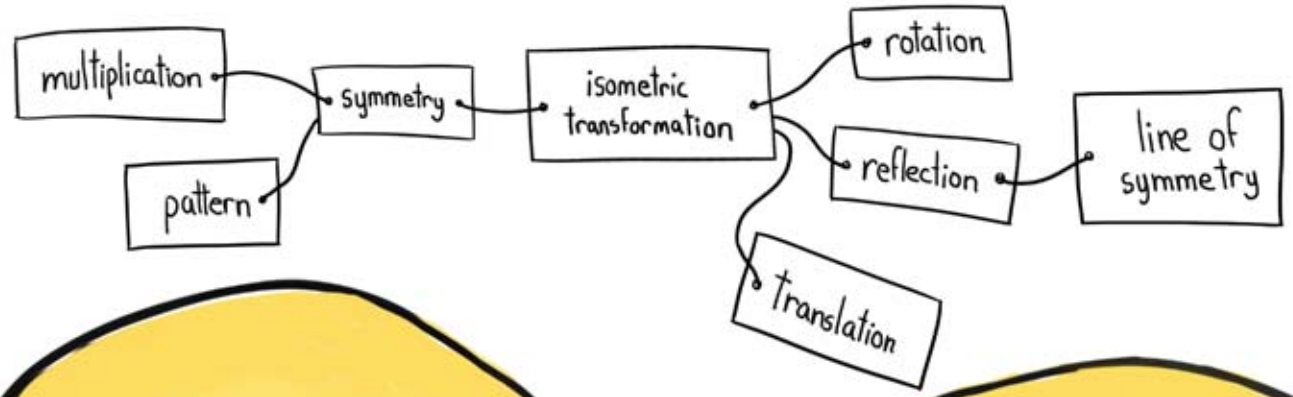
Draw a line across the opening of the mirror book and start closing the book. Your line will form polygons with more and more sides. If you manage to make the opening in the mirror book infinitely small, your polygon will turn into a circle! As with all other ideas that relate to infinity, kids and grown-ups find this deeply meaningful. The idea that polygons approximate circles as the number of sides grows inspired authors of several math adventure books, such as the classic “Flatland” and “The Greedy Triangle.”

Where in science and technology are angles of mirrors important? Help the child change angles of car mirrors. Look inside physical or virtual telescopes and microscopes. Ask a dentist to explain how mirrors help to see inside the mouth. Watch a video about solar sails.



Special snowflake

Fold a piece of paper through its center repeatedly, then sketch and cut out a snowflake. Coffee filters work well for snowflakes. Giant paper is a lot of fun. Children who can't cut yet can draw or point out shapes of cuts.



Make a special snowflake with a child's name. Fold the paper, then write the name in chunky letters that touch one another. Make sure that at least one side of each letter touches the fold.

Make a snowflake out of a toy or an interesting object by tracing its outline on the folded paper. Ensure that at least one side or edge of an object touches the fold.

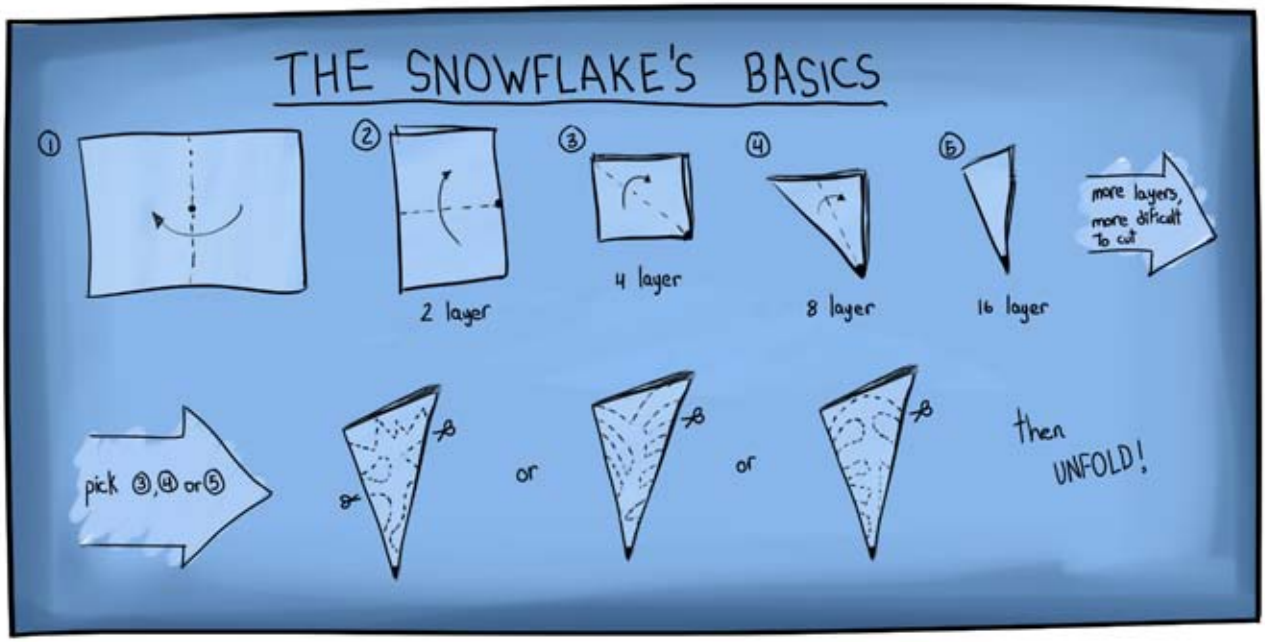
Explore how many times you can fold a piece of paper.

Vary the number of folds and try trickier six – and ten – layer folds. Keep in mind that children frequently forget where the center of the paper was and fold every which way, with unpredictable results. Celebrate their accidental discoveries and figure out new patterns, but also help them achieve patterns they wanted in the first place.

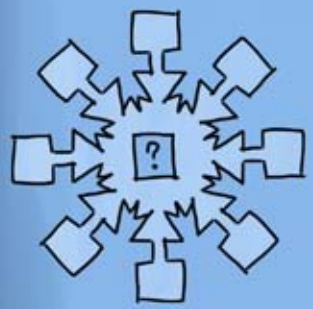
Symmetric patterns fascinate babies. Observe them focus (for two to three seconds at a time or more) as you fold and unfold snowflakes for them. Babies may want to tear up the snowflakes or chew them a bit as a part of the exploration, so be prepared to make new ones quickly!

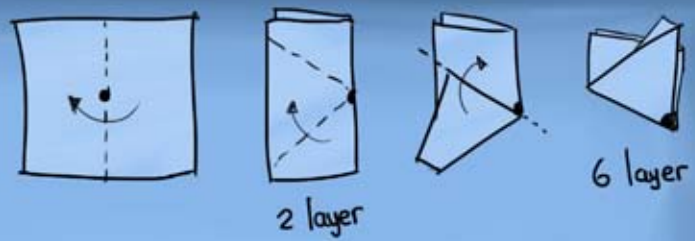
Some toddlers like to guide parents' hands with scissors to design snowflakes, or have parents add hand guidance and pressure to help them with cutting. They can solve and pose simple puzzles, such as what shape each hole will be when you open up the snowflake, or how many holes will each cut make.

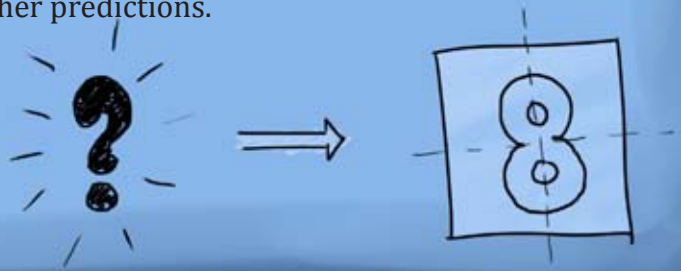
As children continue working with snowflakes, they can make particular shapes appear at different parts of the snowflake (rhombus, oval, flower), figure out different folds, and discuss transformations. They can use snowflakes to study times tables, and make designs for particular multiplication facts (for example, three holes cut into each of eight sectors for three times eight). They can also explore the arts and crafts of symmetric cutting from Oriental, South American, European and other cultures.



Challenge one another to create particular folds or cut out particular shapes. Can you make a square in the middle of the snowflake, or a star at its side?





Make snowflakes with four, five, six, seven, or eight sectors. Use these snowflakes to model times tables.

Try reverse engineering – cut a shape or a letter/number out of unfolded sheet of paper. Let the child explore its line of symmetry (if any) and make predictions as to how folded paper has to be cut to result in the given shape. Test her predictions.